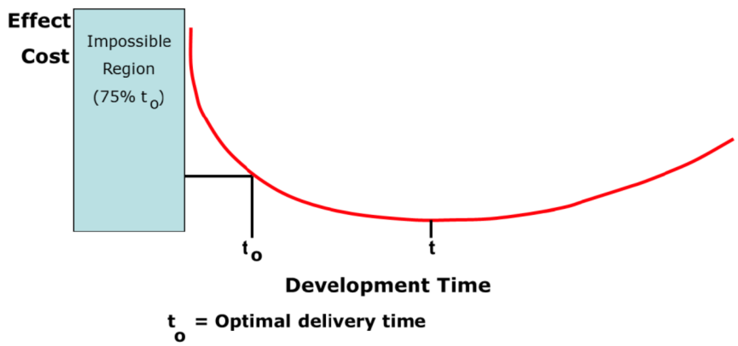
Using the PNR Curve to Convert Effort to Schedule

by Mike Griffiths

For those interested in understanding the relationship between effort and schedule, or anyone asked to complete a project in half the time with double the resources, it is helpful to understand the Putnam-Norden-Rayleigh (PNR) curve. Research by Putnam and Norden found that, for projects that require communication and learning (like software development projects), the effort-to-time curve follows a Rayleigh distribution. Putnam confirmed that this curve applied to software projects in his article "A General Empirical Solution to the Macro Software Sizing and Estimating Problem" (*IEEE Transactions on Software Engineering*, July 1978), and the curve became known as the Putnam-Norden-Rayleigh curve, or the PNR Staffing curve, as shown in the following diagram:



The PNR curve shows time (X axis) against effort and cost (Y axis). As we add more resources to a project, the reduction in effort is not linear. Instead it follows an eversteepening curve. As we move to the left (shortening the project timeline), the curve representing adjusted staff months gets steeper and steeper, indicating increasing costs but not much of a shortening in timeline in comparison to the increase in costs. There are some important points on the curve to understand. The lowest-cost delivery time (shown as the lowest-cost point of the curve, "t") indicates the lowest cost that the project could be delivered for. However, this point does not factor in delayed return on investment, inflation, or work-in-progress costs. Most companies are looking for the best compromise between low cost and short timelines, as indicated by the optimal delivery time ("t_O").

Barry Boehm did an interesting study into attempts to shorten project schedules. He examined over 750 projects that attempted to deliver code in less time than the optimal delivery point (t_O). None of the projects were successful in reducing the schedule below 75 percent of the optimal delivery point (t_O). He christened the area the "Impossible Region" to indicate that you cannot compress schedules beyond this point. (See Barry W. Boehm, *Software Engineering Economics*, Prentice Hall, 1981.)

Intuitively, we know this to be true. As the old saying goes, "You cannot make a baby in one month with nine women." And as Frederick Brooks stated in *The Mythical-Man Month: Essays on Software Engineering* (Addison-Wesley Professional, 1995), "Adding resources to a project that is already late will make it later."

As you approach the Impossible Region, the gradient of the line becomes vertical. In other words, adding resources increases the project spending, but it does not shorten the timeframe. Following Frederick Brooks's observation, I would guess the line might start to curve back to indicate that adding more people not only adds costs, but makes the delivery time longer. And I can see how this would be the case, as decisions and communications become more complex and extra people actually slow things down.

So how do you determine the best compromise between short timeline and low cost (tO)? Barry Boehm provides the following formula in the COCOMO estimation engine:

$$t_O = F * \text{Effort} \wedge 0.33$$

(The effort in person-months cube rooted multiplied by a scaling factor)

In this formula, F is a factor that varies by project type:

- COCOMO II default: 3.67
- Web development: 3.10
- E-commerce development: 3.20
- Military development: 3.80
- Embedded development: 4.00

So imagine we have a web application development project that is estimated to be 28 person-months in duration. If we put 28 into the formula and use the scaling factor for web development (3.1), the calculation would be:

$$t_O = 3.1 * 28 \wedge 0.33 = 9.5 \text{ months}$$

This calculation indicates that a 28 person-month web development project would be optimally implemented in 9.5 months by a team of 28 / 9.5 = 3 people. So 3 people is a good size for a 28 person-month project, but could we get it done in half the time (9.5 /2 = 4.75 months) with 6 people? No, that would take us into the Impossible Region. Understanding the PNR curve is useful to help us validate whether we are being asked to undertake unrealistic plans and to better understand the true cost of adding more and more resources to projects.